



Equation 14 for f involves trial-and-

Although the explicit equations can be solved directly for f, the trial-and-error method converges rapidly.

Numerical technique. The equations for pressure ratio and friction factor are implicit. This requires the use of a numerical method, namely the Newton-Raphson method with a convergence to \pm 0.0001. This requires differentiating the objective function

The Newton-Raphson method is of the form of Equation 15. X_i is the guessed or assumed root of the equation given by F(X) = 0.

 $F(X_i)$ is the value of the objective function. $F'(X_i)$ is the value of the differential of the objective function. The i is the iteration counter, and i_{max} is the maximum iteration.

The derivatives of the objective functions given by Equations 4, 5, and 15, take the form of Equations 16, 17, and 18.

The first assumption, that is, the first iteration in the Newton-Raphson method, must be carefully selected because, sometimes, the root does not converge even after many iterations. The method has been found to converge quadratically.

In this program, default values for the first iteration of f and r are 0.01 and 1.0 respectively. In several problems where the method has been used, these values have been found to converge to yield actual values (roots of the equations) of the friction factor and the pressure ratio after only a few Nomenclature

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A = Pipe internal cross-sectional area, sq ft
D = Pipe ID, sq ft
d = Pipe ID, in.
f = Darcy friction factor
g<sub>c</sub> = Gravitational constant, 32.17 lb<sub>m</sub>·ft/lb<sub>r</sub>·sec<sup>2</sup>
K = Excess head loss for a fitting, velocity heads
K<sub>1</sub> = K for fitting an N<sub>Re</sub> = 1, velocity heads
K<sub>s</sub> = K for very large fittings at N<sub>Re</sub> = ∞, velocity heads
k = ratio of heat capacity at constant pressure to heat capacity at constant volume
Leq = Fittings equivalent length, ft
L = Straight pipe length, ft
L = Total fitting equivalent length plus straight pipe length, ft
M<sub>1</sub> = Mach number at pipe inlet
M<sub>2</sub> = Mach number at pipe outlet
M<sub>3</sub> = Molecular weight of gas
MAPB = Maximum allowable back pressure, psia
n = number of fittings of a given type
N<sub>Re</sub> = Reynolds number
P = P<sub>1</sub> or P<sub>2</sub> depending on input
P<sub>1</sub> = Pipe inlet pressure, psia
P<sub>2</sub> = Pipe outlet pressure, psia
p = Pressure drop, psia
r = Ratio of P<sub>2</sub>/P<sub>1</sub>
t = Absolute temperature, °F. (°F. + 460 = °R.)
V<sub>a</sub> = Actual velocity, fps
V<sub>s</sub> = Sonic velocity, fps
V<sub>s</sub> = Gas flow rate, lb/hr
Z = Gas compressibility factor
e = Absolute roughness of pipe wall, ft (0.00015 for carbon steel pipe)
p = Density, lb/cu ft
μ = Viscosity of gas, lb/ft-hr (cp × 2.42 = lb/ft-hr)
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iterations.

For reference, Gjumbir and Olujic¹⁴ have reviewed nonlinear equations in greater detail.

Pipe losses. An improved method for evaluating head loss caused by fluid rising in elevation, pipe friction factor, and turbulence caused by abrupt changes in direction or cross-sectional area, has been reviewed by Hooper.¹⁵

The method uses two constants plus the Reynolds number and fitting diameter to predict the head loss in an elbow, valve, or tee.

The TWO-K method for calculating the excess head losses or velocity heads for various fittings, namely elbows, valves, and tees, and for exit losses, has been incorporated into the program.

The types of fittings with their corresponding K_1 and K_{∞} values were adapted from Hooper.

The program calculates the K values for individual fittings. It then calculates the total K for all fittings in the pipe system.